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# **Description Logic Programs: Combining Logic Programs with Description Logic**

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*Comment: Gives a fundamental theoretical approach to combining (1.) rules and (2.) ontologies for the Semantic Web, by first defining and focusing on the intersection of (1.) Logic Programs, the core knowledge representation of RuleML, the leading draft standard for Semantic Web rules, and (2.) Description Logic, the core knowledge representation of W3C's OWL, the leading draft standard for Semantic Web ontologies.*

# Description Logic Programs: Combining Logic Programs with Description Logic

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## ABSTRACT

We show how to interoperate, semantically and inferentially, between the leading Semantic Web approaches to rules (RuleML Logic Programs) and ontologies (OWL/DAML+OIL Description Logic) via analyzing their expressive intersection. To do so, we define a new intermediate knowledge representation (KR) contained within this intersection: *Description Logic Programs (DLP)*, and the closely related *Description Horn Logic (DHL)* which is an expressive fragment of first-order logic (FOL). DLP provides a significant degree of expressiveness, substantially greater than the RDF-Schema fragment of Description Logic.

We show how to perform *DLP-fusion*: the bidirectional translation of premises and inferences (including typical kinds of queries) from the DLP fragment of DL to LP, and vice versa from the DLP fragment of LP to DL. In particular, this translation enables one to “build rules on top of ontologies”: it enables the rule KR to have access to DL ontological definitions for vocabulary primitives (e.g., predicates and individual constants) used by the rules. Conversely, the DLP-fusion technique likewise enables one to “build ontologies on top of rules”: it enables ontological definitions to be supplemented by rules, or imported into DL from rules. It also enables available efficient LP inferencing algorithms/implementations to be exploited for reasoning over large-scale DL ontologies.

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Semantic Web, rules, ontologies, logic programs, Description Logic, knowledge representation, XML, RDF, model-theoretic semantics, inferencing, interoperability, translation, information integration

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## 1. INTRODUCTION

The challenge we address in this paper is how and why to combine rules with ontologies for the Semantic Web (SW). In this paper, we focus on meeting key requirements for such a combination of rules and ontologies by establishing the basis for a combined logical knowledge representation (KR) formalism. We start from the current draft standards for ontologies (DAML+OIL) [6] and for rules (RuleML) [3] in the Semantic Web context, and show how aspects of each language can be translated to the other. Both standards correspond with established KR formalisms: Description Logic (DL) in the case of DAML+OIL, and (declarative) logic programs (LP) in the case of RuleML.<sup>1</sup> This correspondence allows us to exploit results w.r.t. the mapping of each KR to classical First Order Logic (FOL).

A mapping between ontology and rule languages is important for many aspects of the Semantic Web:

**Language layering** The Semantic Web can be viewed as largely about “KR meets the Web”. Over the last two years or so, a broad consensus has evolved in the Semantic Web community that the vision of the Semantic Web includes, specifically, rules as well as ontologies. A key requirement for the Semantic Web’s architecture overall, then, is to be able to layer rules on top of ontologies—in particular to create and reason with rule-bases that mention vocabulary specified by ontology-based knowledge bases—and to do so in a semantically coherent and powerful manner.

**Querying** The capabilities of ontology languages with respect to instances can be rather low, and even conjunctive queries—the least expressive query language usually considered in database research—are often not supported [4]. This area is a stronghold of rules, which offer extensive facilities for instance reasoning. Hence, it is interesting to consider combining DLs with the rule paradigm in order to state expressive instance queries w.r.t. terminological knowledge bases.

**Data integration** The majority of today’s data resides in relational databases. As the Semantic Web grows in importance, people will probably start exporting their data according to some chosen ontology. This essentially leads to data that is replicated in order to enable ontology-based processing, e.g., by reading the exported files into a classifier such as FaCT [12] or Racer [10].

<sup>1</sup>In tandem with RuleML/LP, we also focus on the (positive) Horn subset of FOL, which is closely related to the positive Horn subset of LP.

Logic programming systems such as XSB [16], however, can access databases directly through built-in predicates. Furthermore, restricted variants of logic programs, such as the ones established in this paper, can be directly implemented on top of SQL99-compliant relational databases. Hence, an LP-based implementation of an ontology language allows a closer interaction with live data.

**Semantic Web Services** A task-oriented motivation for combining rules with ontologies arises from the efforts to design and build *Semantic Web Services (SWS)*. Semantic Web Services attempt to describe services in a knowledge-based manner in order to use them for a variety of purposes, including: discovery and search; selection, evaluation, negotiation, and contracting; composition and planning; execution; and monitoring. Both rules and ontologies are necessary for such service descriptions and play complementary roles: while ontologies are useful for representing hierarchical categorisation of services overall and of their inputs and outputs, rules are useful for representing contingent features such as business policies, or the relationship between preconditions and post-conditions.

## 2. OVERVIEW OF THE APPROACH

This section gives an overview of our approach and sketches the outline of the remainder of the paper. Our approach is driven by the insight that understanding the expressive *intersection* of two the KR will be crucial to understanding the expressive *combination/union* of the two KR. Hence, we start with the goal of understanding the relationship between both logic based KR formalisms (so as to be able to combine knowledge taken from both): Description Logics (decidable fragments of FOL closely related to propositional modal and dynamic logics [17]), and Logic Programs (see, e.g., [2] for review) which in turn is closely related to the Horn fragment of FOL. Since Description Logics resemble a subset of FOL without function symbols, we similarly focus on the fragment of Horn FOL, *def-Horn*, that contains no function symbols. Both DL and LP are then related to *def-Horn*.

The established correspondence is used to define a new intermediate KR called *Description Horn Logic (DHL)*, which is contained within the intersection, and the closely related *Description Logic Programs (DLP)*, which can be viewed as DHL with a moderate weakening as to the kinds of conclusion can be drawn.

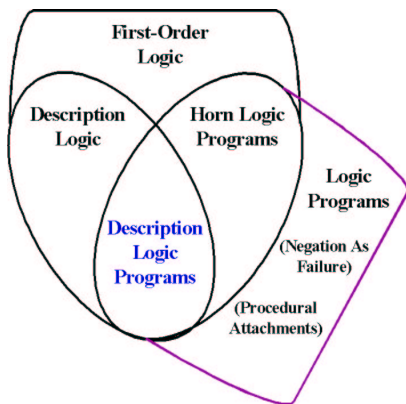


Figure 1: Expressive overlap of DL with LP.

Figure 1 illustrates the relationship between the various KR and their expressive classes. DL and Horn are strict (decidable) subsets of FOL. LP, on the other hand, intersects with FOL but neither includes nor is fully included by FOL. For example FOL can

express (positive) disjunctions, which are inexpressible in LP. On the other hand, several expressive features of LP, which are frequently used in practical rule-based applications, are inexpressible in FOL (and consequently also outside of *def-Horn*). One example is negation-as-failure, a basic kind of logical non-monotonicity. Another example is procedural attachments, e.g., the association of action-performing procedural invocations with the drawing of conclusions about particular predicates.

Description Logic Programs, our newly defined intermediate KR, is contained within the intersection of DL and LP. “Full” LP, including non-monotonicity and procedural attachments, can thus be viewed as including an “ontology sub-language”, namely the DLP subset of DL.

Rather than working from the intersection as we do in this paper, one may instead directly address the expressive union of DL and LP by studying the expressive union of DL and LP within the overall framework of FOL. This is certainly an interesting thing to do. However, to our knowledge, this has not yet been well characterised theoretically, e.g., it is unclear how, if at all, such a union differs from full FOL.

Full FOL has some significant practical and expressive drawbacks as a KR in which to combine DL and rules. First, full FOL has severe computational complexity: it is undecidable in the general case, and intractable even under the Datalog restriction (see Section 3.2). Second, it is not understood even at a basic research level how to expressively extend full FOL to provide non-monotonicity and procedural attachments; yet these are crucial expressive features in many (perhaps most) practical usages of rules. Third, full FOL and its inferencing techniques have severe practicable limitations since it is unfamiliar to the great majority of mainstream software engineers, whereas rules (e.g., in the form of SQL-type queries, or Prolog) are familiar conceptually to many of them.

Via the DLP KR, we give a new technique to combine DL and LP. We show how to perform *DLP-fusion*: the bidirectional mapping of premises and inferences (including typical kinds of queries) from the DLP fragment of DL to LP, and from the DLP fragment of LP to DL. DLP-fusion allows us to fuse the two logical KR so that information from each can be used in the other. The DLP-fusion technique promises several benefits. In particular, DLP-fusion enables one to “build rules on top of ontologies”: it enables the rule KR to have access to DL ontological definitions for vocabulary primitives (e.g., predicates and individual constants) used by the rules. Conversely, the technique enables one to “build ontologies on top of rules”: it enables ontological definitions to be supplemented by rules, or imported into DL from rules. It also enables efficient LP inferencing algorithms/implementations, e.g., rule or relational database engines, to be exploited for reasoning over large-scale DL ontologies.

## 3. PRELIMINARIES

In this section we will introduce Horn Logic, Description Logic (DL) and the DL based ontology language DAML+OIL. In particular, we will describe their syntax and formalise their meaning in terms of classical First Order Logic (FOL).

### 3.1 DAML+OIL and Description Logic

DAML+OIL is an ontology language designed for use on the (semantic) web. Although DAML+OIL is syntactically “layered” on top of RDFS, semantically it is layered on a subset of RDFS. This subset does *not* include RDFS’s recursive meta model (i.e., the unrestricted use of the type relation), but instead treats RDFS

Constructor	DL Syntax	Example
intersectionOf	$C_1 \sqcap \dots \sqcap C_n$	Human $\sqcap$ Male
unionOf	$C_1 \sqcup \dots \sqcup C_n$	Doctor $\sqcup$ Lawyer
complementOf	$\neg C$	$\neg$ Male
oneOf	$\{i_1 \dots i_n\}$	{john, mary}
hasClass	$\exists P.C$	$\exists$ hasChild.Lawyer
toClass	$\forall P.C$	$\forall$ hasChild.Doctor
hasValue	$\exists P.\{i\}$	$\exists$ citizenOf.{USA}
minCardinalityQ	$\geq n P.C$	$\geq 2$ hasChild.Lawyer
maxCardinalityQ	$\leq n P.C$	$\leq 1$ hasChild.Male
cardinalityQ	$= n P.C$	$= 1$ hasParent.Female

Figure 3: DAML+OIL class constructors

as a very simple DL supporting only atomic class names. Like other DLs, this “DAML+OIL subset” of RDFS corresponds to a fragment of classical FOL, making it much easier to develop mappings to rule languages as well as to DLs. From now on, when we talk about RDFS, we will be referring to the DAML+OIL subset of RDFS.

DAML+OIL is equivalent to a very expressive DL—in fact it is equivalent to the  $SHOIQ(\mathbf{D})$  DL [13, 11]. In addition to “abstract” classes and individuals, DAML+OIL also supports the use of “concrete” datatypes and data values (the  $(\mathbf{D})$  in  $SHOIQ(\mathbf{D})$ ). In this paper, however, we will restrict our attention to the abstract part of the language, which corresponds to the  $SHOIQ$  DL.

Figure 2 (on page ) shows how DAML+OIL statements correspond to  $SHOIQ$  axioms, where  $C$  (possibly subscripted) is a class,  $P$  (possibly subscripted) is a property,  $P^-$  is the inverse of  $P$ ,  $P^+$  is the transitive closure of  $P$ ,  $i$  (possibly subscripted) is an individual and  $\top$  is an abbreviation for  $A \sqcup \neg A$  for some class  $A$  (i.e., the most general class, called “Thing” in DAML+OIL).

It can be seen that all DAML+OIL statements can be reduced to class/property inclusion axioms and ground facts (asserted class-instance and instance-property-instance relationships).<sup>2</sup> In the case of `transitiveProperty`, however, the axiom  $P^+ \sqsubseteq P$  is taken to be equivalent to asserting that  $P$  is a transitive property (like DAML+OIL,  $SHOIQ$  does not include the transitive closure operator).

As in any DL, DAML+OIL classes can be names (URIs) or *expressions*, and a variety of *constructors* are provided for building class expressions. Figure 3 summarises the available constructors and their correspondence with  $SHOIQ$  class expressions.

The meaning of  $SHOIQ$  is usually given by a model theory [13]. However,  $SHOIQ$  can also be seen in terms of a correspondence to FOL, where classes correspond to unary predicates, properties correspond to binary predicates and subclass/property axioms correspond to implication [7, 4].

To be more precise, individuals are equivalent to FOL constants, classes and class expressions are equivalent to FOL formulae with one free variable, and properties (and property expressions when supported by the DL) are equivalent to FOL formulae with two free variables. Class and property inclusion axioms are equivalent to FOL sentences consisting of an implication between two formulae with the free variables universally quantified at the outer level. E.g., a DL axiom of the form  $C \sqsubseteq D$  is equivalent to a FOL sentence of the form  $\forall x.C(x) \rightarrow D(x)$ . DL axioms of the form  $a : C$  and  $\langle a, b \rangle : P$  correspond to ground atoms  $C(a)$  and  $P(a, b)$ . Finally, DL axioms asserting the transitivity of a property  $P$ , the functionality of a property  $P$  and that property  $Q$  is the inverse of property

<sup>2</sup>Equivalence axioms can be reduced to a symmetrical pair of inclusion axioms.

DL	FOL
$a : C$	$C(a)$
$\langle a, b \rangle : P$	$P(a, b)$
$C \sqsubseteq D$	$\forall x.C(x) \rightarrow D(x)$
$P^+ \sqsubseteq P$	$\forall x, y, z.(P(x, y) \wedge P(y, z)) \rightarrow P(x, z)$
$\top \sqsubseteq \leq 1 P$	$\forall x, y, z.(P(x, y) \wedge P(x, z)) \rightarrow y = z$
$P \equiv Q^-$	$\forall x, y.P(x, y) \iff Q(y, x)$
$C_1 \sqcap \dots \sqcap C_n$	$C_1(x) \wedge \dots \wedge C_n(x)$
$C_1 \sqcup \dots \sqcup C_n$	$C_1(x) \vee \dots \vee C_n(x)$
$\neg C$	$\neg C(x)$
$\{a_1, \dots, a_n\}$	$x = a_1 \vee \dots \vee x = a_n$
$\exists P.C$	$\exists y.(P(x, y) \wedge C(y))$
$\forall P.C$	$\forall y.(P(x, y) \rightarrow C(y))$
$\geq n P.C$	$\exists y_1, \dots, y_n. \bigwedge_{1 \leq i \leq n} (P(x, y_i) \wedge C(y_i))$
$\leq (n-1) P.C$	$\forall y_1, \dots, y_n. (\bigwedge_{1 \leq i \leq n} (P(x, y_i) \wedge C(y_i)) \rightarrow (\bigvee_{1 \leq i < n, i < j \leq n} y_i = y_j))$

Figure 4: DL FOL equivalence

$P$  are equivalent to FOL sentences of the form  $\forall x, y, z.(P(x, y) \wedge P(y, z)) \rightarrow P(x, z)$ ,  $\forall x, y, z.(P(x, y) \wedge P(x, z)) \rightarrow y = z$  and  $\forall x, y.P(x, y) \iff Q(y, x)$  respectively.

Figure 4 summarises the above equivalences and shows the FOL formulae corresponding to the DL class expressions described in Figure 3, where  $a, b$  are constants, and  $x$  is the free variable. These formulae can be composed in the obvious way, e.g.,  $\exists R.(C \sqcap D) \equiv \exists y.(P(x, y) \wedge (C(y) \wedge D(y)))$ .

As a notational convention we will, throughout the paper, use  $a$  and  $b$  for constants and  $w, x, y$  and  $z$  for variables.

### 3.2 Logic Programs and Horn FOL

Declarative logic programs (LP) is the KR whose semantics underlies in a large part the four families of rule systems that are currently most commercially important—SQL relational databases, OPS5-heritage production rules, Prolog, and Event-Condition-Action rules—as well as the proposals for rules in context of the Semantic Web.

As mentioned before, the commonly used expressiveness of full LP includes features, notably negation-as-failure/priorities and procedural attachments, that are not expressible in FOL, much less in DL. We thus concentrate on only an expressive portion of LP.

An *ordinary* (a.k.a. “normal”<sup>3</sup>) LP is a set of *rules* each having the form:

$$H \leftarrow B_1 \wedge \dots \wedge B_m \wedge \sim B_{m+1} \wedge \dots \wedge \sim B_n$$

where  $H, B_i$  are atoms (atomic formulae), and  $n \geq m \geq 0$ . Note that no restriction is placed on the arity of the predicates appearing in these atoms. Logical variables, and logical functions (with any arity), may appear unrestrictedly in these atoms.

$H$  is called the *head* (a.k.a. *consequent*) of the rule;

$$B_1 \wedge \dots \wedge B_m \wedge \sim B_{m+1} \wedge \dots \wedge \sim B_n$$

is called the *body* (a.k.a. *antecedent*) of the rule.  $\leftarrow$  is to be read as “if”, so that the overall rule should be read as “[head] if [body]”, i.e., “if [body] then [head]”. If  $n = 0$ , then the body is empty, i.e., *True*, and notationally the “ $\leftarrow$ ” is often omitted. A *fact* is a rule whose body is empty and whose head is a ground atom.  $\sim$  stands for negation-as-failure, a logically non-monotonic form of negation whose semantics differs, in general, significantly from the

<sup>3</sup>[2] call this “general”; however, there are actually a number of frequently used extensions!

Axiom	DL Syntax	Example
subClassOf	$C_1 \sqsubseteq C_2$	Human $\sqsubseteq$ Animal $\sqcap$ Biped
sameClassAs	$C_1 \equiv C_2$	Man $\equiv$ Human $\sqcap$ Male
subPropertyOf	$P_1 \sqsubseteq P_2$	hasDaughter $\sqsubseteq$ hasChild
samePropertyAs	$P_1 \equiv P_2$	cost $\equiv$ price
disjointWith	$C_1 \sqsubseteq \neg C_2$	Male $\sqsubseteq \neg$ Female
sameIndividualAs	$\{i_1\} \equiv \{i_2\}$	{President_Bush} $\equiv$ {G.W_Bush}
differentIndividualFrom	$\{i_1\} \sqsubseteq \neg\{i_2\}$	{john} $\sqsubseteq \neg$ {peter}
inverseOf	$P_1 \equiv P_2^-$	hasChild $\equiv$ hasParent <sup>-</sup>
transitiveProperty	$P^+ \sqsubseteq P$	ancestor <sup>+</sup> $\sqsubseteq$ ancestor
uniqueProperty	$\top \sqsubseteq \leq 1 P . \top$	$\top \sqsubseteq \leq 1$ hasMother. $\top$
unambiguousProperty	$\top \sqsubseteq \leq 1 P^- . \top$	$\top \sqsubseteq \leq 1$ isMotherOf <sup>-</sup> . $\top$
range	$\top \sqsubseteq \forall P . C$	$\top \sqsubseteq \forall$ hasParent.Human
domain	$\top \sqsubseteq \forall P^- . C$	$\top \sqsubseteq \forall$ hasParent <sup>-</sup> .Human
$i$ type $C$	$i : C$	john : Man
$i_1 P i_2$	$\langle i_1, i_2 \rangle : P$	$\langle$ john, peter $\rangle :$ hasParent

Figure 2: DAML+OIL statements and SHOIQ axioms

semantics of classical negation ( $\neg$ ). Intuitively,  $\sim B_i$  means “ $B_i$  is not believed” (i.e., is unknown or false), whereas  $\neg$  means “ $B_i$  is false”. Intuitively, each rule can be viewed as universally quantified at the outer level. More precisely, each rule can be viewed as standing for the set of all its ground instantiations.

A *definite* LP is an ordinary LP in which negation-as-failure does not appear, i.e., a set of rules each having the form:

$$H \leftarrow B_1 \wedge \dots \wedge B_m$$

where  $H, B_i$  are atoms, and  $m \geq 0$ .

Definite LP is closely related syntactically and semantically to the Horn fragment of FOL, a.k.a. Horn-clause logic. A clause in FOL has the form:

$$L_1 \vee \dots \vee L_k$$

where each  $L_i$  is a (classical) literal. A literal  $L$  has either the form (1)  $A$  or (2)  $\neg A$ , where  $A$  is an atom. The literal is said to be *positive* in case (1), or to be *negative* in case (2). A clause is said to be *Horn* when *at most one* of its literals is positive. A Horn clause is said to be *definite* when *exactly one* of its literals is positive. A definite Horn clause is also known as a *Horn rule*. A definite Horn clause, a.k.a. *Horn rule*, can thus be written in the form:

$$H \leftarrow B_1 \wedge \dots \wedge B_m$$

where  $H, B_i$  are atoms, and  $m \geq 0$ . We say that this Horn rule *corresponds* to the definite LP rule that has the same syntactic form, and vice versa. Likewise, we say that a Horn ruleset  $\mathcal{RH}$  and a definite LP ruleset  $\mathcal{RP}$  correspond to each other when their rules do (isomorphically). We then also say that  $\mathcal{RP}$  is the *LP-correspondent* of  $\mathcal{RH}$ , and conversely that  $\mathcal{RH}$  is the *Horn-correspondent* of  $\mathcal{RP}$ .

As mentioned above, it is implicit in this notation that all logical variables are universally quantified at the outer level, i.e., over the scope of the whole clause. E.g., the rule

$$man(x) \leftarrow human(x) \wedge male(x)$$

can be written equivalently as:

$$\forall x. man(x) \leftarrow human(x) \wedge male(x).$$

Note the similarity with the FOL equivalent of a DL inclusion (subClassOf) axiom given in Figure 4.

An LP rule or Horn clause is said to be *equality-free* when the equality predicate does not appear in it. Likewise, each is said to be *Datalog* when no logical functions (of arity greater than zero) appear in it.<sup>4</sup>

The semantics of an ordinary LP is defined to be a *conclusion set*, where each conclusion is a ground atom, i.e., fact, entailed by the LP. Formally, the semantics of a definite LP  $\mathcal{R}$  is defined as follows. Let HB stand for the Herbrand base of  $\mathcal{R}$ . The conclusion set  $\mathcal{C}$  is the smallest (w.r.t. set inclusion) subset  $\mathcal{S}$  of HB such that for any rule

$$H \leftarrow B_1 \wedge \dots \wedge B_m,$$

if  $B_1 \wedge \dots \wedge B_m \in \mathcal{S}$  then  $H \in \mathcal{S}$ .

The relationship of LP semantics to FOL semantics is relatively simple to describe for the case of definite equality-free Datalog LP, which we call *def-LP*. The syntactically corresponding fragment of FOL is definite equality-free Datalog Horn FOL, which we call *def-Horn*. Let  $\mathcal{RP}$  be a def-LP. Let  $\mathcal{RH}$  stand for the corresponding def-Horn ruleset. The conclusion set of  $\mathcal{RP}$  then coincides with the minimal (w.r.t. set inclusion) Herbrand model of  $\mathcal{RH}$ .

Hence, the def-LP and the def-Horn ruleset entail exactly the same set of facts. Every conclusion of the def-LP is also a conclusion of the def-Horn ruleset. Relative to the def-Horn ruleset, the def-LP is thus sound; moreover, it is complete for fact-form conclusions, i.e., for queries whose answers amount to conjunctions of facts. However, the def-LP is a mildly *weaker* version of the def-Horn ruleset, in the following sense. Every conclusion of the def-LP must have the form of a fact. By contrast, the entailments, i.e., conclusions, of the def-Horn ruleset are not restricted to be facts. E.g., suppose  $\mathcal{RH}$  consists of the two rules

$$kiteDay(Tues) \leftarrow sunny(Tues) \wedge windy(Tues)$$

and

$$sunny(Tues).$$

Then it entails

$$kiteDay(Tues) \leftarrow windy(Tues)$$

(a non-unit derived clause) whereas  $\mathcal{RP}$  does not. In practical applications, however, quite often only the fact-form conclusions are desired, e.g., an application might be interested above only in whether or not *kiteDay(Tues)* is entailed. The def-LP has the virtue of conceptual and computational simplicity. Thinking in terms of expressive classes, we will view def-LP as an *expressive subset* of def-Horn—we will call it the expressive *f-subset*. def-LP is a mild weakening of def-Horn along the dimension of entailment power, permitting only fact-form conclusions—we will call this *f-weakening*.

In return for this f-weakening, def-LP has some quite attractive (in the body).

<sup>4</sup>The Datalog restriction is usually taken to mean, additionally, no negation and only “safe” rules (all variables in the head also occur

computational characteristics (as well as being expressively extensible in directions that FOL is not, as discussed earlier). For the propositional case of def-LP, exhaustive inferencing is  $O(n)$  where  $n = |\mathcal{R}\mathcal{P}|$  — i.e., worst-case linear time [8]. For the general case with logical variables, the entire conclusion set of a def-LP  $\mathcal{R}\mathcal{P}$  can be computed in time  $O(n^{v+1})$ , when there is a constant bound  $v$  on the number of logical variables per rule (this restriction, which we will call *VB*, is typically met in practise). Inferencing in def-LP is thus tractable (worst-case polynomial time) given *VB*. In contrast, DLs are generally not tractable (typically ExpTime or even NExp-Time complexity for key inference problems), and full FOL is not decidable.

## 4. MAPPING DL TO def-Horn

In this section we will discuss how DL languages (e.g., DAML+OIL) can be mapped to def-Horn, and vice versa.

### 4.1 Expressive Restrictions

We will first discuss the expressive restrictions of DL and def-Horn as these will constrain the subset of DL and def-Horn for which a complete mapping can be defined.

DLs are decidable subsets of FOL where the decidability is due in large part to their having (a form of) the tree model property [19].<sup>5</sup> This property says that a DL class  $C$  has a model (an interpretation  $\mathcal{I}$  in which  $C^{\mathcal{I}}$  is non-empty) iff  $C$  has a tree-shaped model, i.e., one in which the interpretation of properties defines a tree shaped directed graph.

This requirement severely restricts the way variables and quantifiers can be used. In particular, quantifiers must be *relativised* via atomic formulae (as in the guarded fragment of FOL [9]), i.e., the quantified variable must occur in a property predicate along with the free variable (recall that DL classes correspond to formulae with one free variable). For example, the DL class  $\exists P.C$  corresponds to the FOL formula  $\exists y.(P(x, y) \wedge C(y))$ , where the property predicate  $P$  acts as a guard. One obvious consequence of this restriction is that it is impossible to describe classes whose instances are related to another anonymous individual via different property paths. For example, it is impossible to assert that individuals who live and work at the same location are “HomeWorkers”. This is easy with a Horn rule, e.g.:

$$\text{HomeWorker}(x) \leftarrow \text{work}(x, y) \wedge \text{live}(x, z) \wedge \text{loc}(y, w) \wedge \text{loc}(z, w)$$

Another restriction in DLs is that only unary and binary predicates can usually be captured.<sup>6</sup> This is a less onerous restriction, however, as techniques for reifying higher arity predicates are well known [12].

Definite Horn FOL requires that all variables are universally quantified (at the outer level of the rule), and restricts logical connectives in certain ways. One obvious consequence of the restriction on quantifiers is that it is impossible to assert the existence of individuals whose identity might not be known. For example, it is impossible to assert that all persons have a father (known or unknown). This is easy with a DL axiom, e.g.:

$$\text{Person} \sqsubseteq \exists \text{father}. \top$$

<sup>5</sup>Expressive features such as transitive properties and the `oneOf` constructor compromise the tree model property to some extent, e.g., transitive properties can cause “short-cuts” down branches of the tree.

<sup>6</sup>This is not an inherent restriction, and n-ary DLs are known, e.g.,  $\mathcal{DLR}$  [5].

No negation may appear within the body of a rule, nor within the head. No existentials may appear within the head. Thus it is impossible to assert, e.g., that all persons are either men or women (but not both). This would also be easy using DL axioms, e.g.:

$$\begin{aligned} \text{Person} &\sqsubseteq \text{Man} \sqcup \text{Woman} \\ \text{Man} &\sqsubseteq \neg \text{Woman}. \end{aligned}$$

The Datalog restriction of def-Horn is not an issue for mapping DL into it, since DL also has the Datalog restriction. Finally, the equality-free restriction of def-Horn is a significant restriction in that it prevents representing (partial-)functionality of a property and also prevents representing maximum cardinality. The prohibition against existentials in the head prevents representing minimum cardinality.

### 4.2 Mapping Statements

In this section, we show how (some of) the *statements* (axioms) of DL and DL based languages (such as DAML+OIL and OWL) correspond to def-Horn statements (rules).

#### 4.2.1 RDFS Statements

RDFS provides a subset of the DL statements described in Section 3.1: subclass, subproperty, range, and domain statements (which in a DL setting are often called Tbox axioms); and asserted class-instance (type) and instance-property-instance relationships (which in a DL setting are often called Abox axioms).

As we saw in Section 3.1, a DL inclusion axiom corresponds to an FOL implication. This leads to a straightforward mapping from class and property inclusion axioms to def-Horn rules as follows:

$$\begin{aligned} C \sqsubseteq D, \text{ i.e., class } C \text{ is subclass of class } D, \text{ maps to:} \\ D(x) \leftarrow C(x) \end{aligned}$$

$$\begin{aligned} Q \sqsubseteq P, \text{ i.e., } Q \text{ is a subproperty of } P, \text{ maps to:} \\ P(x, y) \leftarrow Q(x, y) \end{aligned}$$

As shown in Figure 2, RDFS range and domain statements correspond to DL axioms of the form  $\top \sqsubseteq \forall P.C$  (range of  $P$  is  $C$ ) and  $\top \sqsubseteq \forall P^-.C$  (domain of  $P$  is  $C$ ). From Figure 4, we can see that these are equivalent to the FOL sentences  $\forall x. \text{true} \rightarrow (\forall y. P(x, y) \rightarrow C(y))$  and  $\forall x. \text{true} \rightarrow (\forall y. P(y, x) \rightarrow C(y))$ , which can be simplified to  $\forall x, y. P(x, y) \rightarrow C(y)$  and  $\forall x, y. P(y, x) \rightarrow C(y)$  respectively. These FOL sentences are already in def-Horn form, which gives us the following mappings for range and domain:

$$\begin{aligned} \top \sqsubseteq \forall P.C, \text{ i.e., the range of property } P \text{ is class } C, \text{ maps to:} \\ C(y) \leftarrow P(x, y) \end{aligned}$$

$$\begin{aligned} \top \sqsubseteq \forall P^-.C, \text{ i.e., the domain of property } P \text{ is class } C, \text{ maps to:} \\ C(y) \leftarrow P(y, x) \end{aligned}$$

Finally, asserted class-instance (type) and instance-property-instance relationships, which correspond to DL axioms of the form  $a : C$  and  $\langle a, b \rangle : P$  respectively (Abox axioms), are equivalent to FOL sentences of the form  $C(a)$  and  $P(a, b)$ , where  $a$  and  $b$  are constants. These are already in def-Horn form: they are simply rules with empty bodies (which are normally omitted):

$$\begin{aligned} a : C, \text{ i.e., the individual } a \text{ is an instance of the class } C, \\ \text{maps to:} \\ C(a) \end{aligned}$$

$$\begin{aligned} \langle a, b \rangle : P, \text{ i.e., the individual } a \text{ is related to the individual } b \\ \text{via the property } P, \text{ maps to:} \\ P(a, b) \end{aligned}$$

Note that in these rules  $a$  and  $b$  are ground (constants).

### 4.2.2 DAML+OIL statements

DAML+OIL extends RDF with additional statements about classes and properties (Tbox axioms). In particular, it adds explicit statements about class, property and individual equality and inequality, as well as statements asserting property inverses, transitivity, functionality (unique) and inverse functionality (unambiguous).

As discussed in Section 3.1, class and property equivalence axioms can be replaced with a symmetrical pair of inclusion axioms, so they can be mapped to a symmetrical pair of def-Horn rules as follows:

$C \equiv D$ , i.e., the class  $C$  is equivalent to (has the same extension as) the class  $D$ , maps to:

$$\begin{aligned} D(x) &\leftarrow C(x) \\ C(x) &\leftarrow D(x) \end{aligned}$$

$P \equiv Q$ , i.e., the property  $P$  is equivalent to (has the same extension as) the property  $Q$ , maps to:

$$\begin{aligned} Q(x, y) &\leftarrow P(x, y) \\ P(x, y) &\leftarrow Q(x, y) \end{aligned}$$

As we saw in Section 3.1, the semantics of inverse axioms of the form  $P \equiv Q^-$  are captured by FOL sentences of the form  $\forall x, y. P(x, y) \iff Q(x, y)$ , and the semantics of transitivity axioms of the form  $P^+ \sqsubseteq P$  are captured by FOL sentences of the form  $\forall x, y, z. P(x, y) \wedge P(y, z) \rightarrow P(x, z)$ . This leads to a direct mapping into def-Horn as follows:

$P \equiv Q^-$ , i.e., the property  $Q$  is the inverse of the property  $P$ , maps to:

$$\begin{aligned} Q(y, x) &\leftarrow P(x, y) \\ P(x, y) &\leftarrow Q(y, x) \end{aligned}$$

$P^+ \sqsubseteq P$ , i.e., the property  $P$  is transitive, maps to:

$$P(x, z) \leftarrow P(x, y) \wedge P(y, z)$$

As we saw in Section 3.1, DL axioms asserting the functionality of properties correspond to FOL sentences with equality. E.g., a DL axiom  $\top \sqsubseteq \leq 1 P$  ( $P$  is a functional property) corresponds to the FOL sentence  $\forall x, y, z. P(x, y) \wedge P(x, z) \rightarrow y = z$ .<sup>7</sup> This kind of axiom cannot be dealt with in our current framework (see Section 4.1) as it would require def-Horn rules with equality in the head, i.e., rules of the form  $(y = z) \leftarrow P(x, y) \wedge P(x, z)$ .

## 4.3 Mapping Class Constructors

In the previous section we showed how DL axioms correspond with def-Horn rules, and how these can be used to make statements about classes and properties. In DLs, the classes appearing in such axioms need not be atomic, but can be complex compound expressions built up from atomic classes and properties using a variety of constructors. A great deal of the power of DLs derives from this feature, and in particular from the set of constructors provided.<sup>8</sup> In the following section we will show how these DL expressions correspond to expressions in the body of def-Horn rules.

In the following we will, as usual, use  $C, D$  to denote classes,  $P, Q$  to denote properties and  $n$  to denote an integer.

### Conjunction (DL $\sqcap$ )

A DL class can be formed by conjoining existing classes, e.g.,  $C \sqcap D$ . From Figure 4 it can be seen that this corresponds to a

<sup>7</sup>Note that, technically, this is partial-functionality as for any given  $x$  there is no requirement that there exist a  $y$  such that  $P(x, y)$ .

<sup>8</sup>Note that this feature is not supported in the RDFS subset of DLs.

conjunction of unary predicates. Conjunction can be directly expressed in the body of a def-Horn rule. E.g., when a conjunction occurs on the l.h.s. of a subclass axiom, it simply becomes conjunction in the body of the corresponding rule

$$C_1 \sqcap C_2 \sqsubseteq D \equiv D(x) \leftarrow C_1(x) \wedge C_2(x)$$

Similarly, when a conjunction occurs on the r.h.s. of a subclass axiom, it becomes conjunction in the head of the corresponding rule:

$$C \sqsubseteq D_1 \sqcap D_2 \equiv D_1(x) \wedge D_2(x) \leftarrow C(x),$$

This is then easily transformed (via the Lloyd-Topor transformations [14]) into a pair of def-Horn rules:

$$\begin{aligned} D_1(x) &\leftarrow C(x) \\ D_2(x) &\leftarrow C(x) \end{aligned}$$

### Disjunction (DL $\sqcup$ )

A DL class can be formed from a disjunction of existing classes, e.g.,  $C \sqcup D$ . From Figure 4 it can be seen that this corresponds to a disjunction of unary predicates. When a disjunction occurs on the l.h.s. of a subclass axiom it simply becomes disjunction in the body of the corresponding rule:

$$C_1 \sqcup C_2 \sqsubseteq D \equiv D(x) \leftarrow C_1(x) \vee C_2(x)$$

This is easily transformed (again by Lloyd-Topor) into a pair of def-Horn rules:

$$\begin{aligned} D(x) &\leftarrow C_1(x) \\ D(x) &\leftarrow C_2(x) \end{aligned}$$

When a disjunction occurs on the r.h.s. of a subclass axiom it becomes a disjunction in the head of the corresponding rule, and this cannot be handled within the def-Horn framework.

### Universal Restriction (DL $\forall$ )

In a DL, the universal quantifier can only be used in *restrictions*—expressions of the form  $\forall P.C$  (see Section 4.1). This is equivalent to an FOL clause of the form  $\forall y. P(x, y) \rightarrow C(y)$  (see Figure 4).  $P$  must be a single primitive property, but  $C$  may be a compound expression. Therefore, when a universal restriction occurs on the r.h.s. of a subclass axiom it becomes an implication in the head of the corresponding rule:

$$C \sqsubseteq \forall P.D \equiv (D(y) \leftarrow P(x, y)) \leftarrow C(x),$$

which is easily transformed into the standard def-Horn rule:

$$D(y) \leftarrow C(x) \wedge P(x, y).$$

When a universal restriction occurs on the l.h.s. of a subclass axiom it becomes an implication in the body of the corresponding rule. This cannot, in general, be mapped into def-Horn as it would require negation in a rule body.

### Existential Restriction (DL $\exists$ )

In a DL, the existential quantifier (like the universal quantifier) can only be used in restrictions of the form  $\exists P.C$ . This is equivalent to an FOL clause of the form  $\exists y. P(x, y) \wedge C(y)$  (see Figure 4).  $P$  must be a single primitive property, but  $C$  may be a compound expression.

When an existential restriction occurs on the l.h.s. of a subclass axiom, it becomes a conjunction in the body of a standard def-Horn rule:

$$\exists P.C \sqsubseteq D \equiv D(x) \leftarrow P(x, y) \wedge C(y).$$



When an existential restriction occurs on the r.h.s. of a subclass axiom, it becomes a conjunction in the head of the corresponding rule, with a variable that is existentially quantified. This cannot be handled within the def-Horn framework.

#### Negation and Cardinality Restrictions (DL $\neg$ , $\geq$ and $\leq$ )

These constructors cannot, in general, be mapped into def-Horn. The case of negation is obvious as negation is not allowed in either the head or body of a def-Horn rule. As can be seen in Figure 4, cardinality restrictions correspond to assertions of variable equality and inequality in FOL, and this is again outside of the def-Horn framework.

In some cases, however, it would be possible to simplify the DL expression using the usual rewriting tautologies of FOL in order to eliminate the offending operator(s). For example, negation can always be pushed inward by using a combination of De Morgan's laws and equivalences such as  $\neg\exists P.C \equiv \forall P.\neg C$  and  $\neg\geq n P \equiv \leq (n-1) P$  [1]. Further simplifications are also possible, e.g., using the equivalences  $C \sqcup \neg C \equiv \top$ , and  $\forall P.\top \equiv \top$ . For the sake of simplicity, however, we will assume that DL expressions are in a canonical form where all relevant simplifications have been carried out.

#### 4.4 Defining DHL via a Recursive Mapping from DL to def-Horn

As we saw in Section 4.3, some DL constructors (conjunction and universal restriction) can be mapped to the heads of rules whenever they occur on the r.h.s. of an inclusion axiom, while some DL constructors (conjunction, disjunction and existential restriction) can be mapped to the bodies of rules whenever they occur on the l.h.s. of an inclusion axiom. This naturally leads to the definition of two DL languages, classes from which can be mapped into the head or body of LP rules; we will refer to these two languages as  $\mathcal{L}_h$  and  $\mathcal{L}_b$  respectively.

The syntax of the two languages is defined as follows. In both languages an atomic name  $A$  is a class, and if  $C$  and  $D$  are classes, then  $C \sqcap D$  is also a class. In  $\mathcal{L}_h$ , if  $C$  is a class and  $R$  is a property, then  $\forall R.C$  is also a class, while in  $\mathcal{L}_b$ , if  $D, C$  are classes and  $R$  is a property, then  $C \sqcup D$  and  $\exists R.C$  are also classes.

Using the mappings from Section 4.3, we can now follow the approach of [4] and define a recursive mapping function  $T$  which takes a DL axiom of the form  $C \sqsubseteq D$ , where  $C$  is an  $\mathcal{L}_b$ -class and  $D$  is an  $\mathcal{L}_h$ -class, and maps it into an LP rule of the form  $A \leftarrow B$ . The mapping is defined as follows:

$$\begin{aligned} T(C \sqsubseteq D) &\longrightarrow Th(D, y) \leftarrow Tb(C, y) \\ Th(A, x) &\longrightarrow A(x) \\ Th((C \sqcap D), x) &\longrightarrow Th(C, x) \wedge Th(D, x) \\ Th((\forall R.C), x) &\longrightarrow Th(C, y) \leftarrow R(x, y) \\ Tb(A, x) &\longrightarrow A(x) \\ Tb((C \sqcap D), x) &\longrightarrow Tb(C, x) \wedge Tb(D, x) \\ Tb((C \sqcup D), x) &\longrightarrow Tb(C, x) \vee Tb(D, x) \\ Tb((\exists R.C), x) &\longrightarrow R(x, y) \wedge Tb(C, y) \end{aligned}$$

where  $A$  is an atomic class name,  $C$  and  $D$  are classes,  $R$  is a property and  $x, y$  are variables, with  $y$  being a ‘‘fresh’’ variable, i.e., one that has not previously been used.

As we saw in Section 4.3, rules of the form  $(H \wedge H') \leftarrow B$  are rewritten as two rules  $H \leftarrow B$  and  $H' \leftarrow B$ ; rules of the form  $(H \leftarrow H') \leftarrow B$  are rewritten as  $H \leftarrow (B \wedge H')$ ; and rules of the form  $H \leftarrow (B \vee B')$  are rewritten as two rules  $H \leftarrow B$  and  $H \leftarrow B'$ .

For example,  $T$  would map the DL axiom

$$A \sqcap \exists R.C \sqsubseteq B \sqcap \forall P.D$$

into the LP rule

$$B(x) \wedge (D(z) \leftarrow P(x, z)) \leftarrow A(x) \wedge R(x, y) \wedge C(x)$$

which is rewritten as the pair of rules

$$\begin{aligned} B(x) &\leftarrow A(x) \wedge R(x, y) \wedge C(x) \\ D(z) &\leftarrow A(x) \wedge R(x, y) \wedge C(x) \wedge P(x, z). \end{aligned}$$

We call  $\mathcal{L}$  the intersection of  $\mathcal{L}_h$  and  $\mathcal{L}_b$ , i.e., the language where an atomic name  $A$  is a class, and if  $C$  and  $D$  are classes, then  $C \sqcap D$  is also a class. We then extend  $T$  to deal with axioms of the form  $C \equiv D$ , where  $C$  and  $D$  are both  $\mathcal{L}$ -classes:

$$T(C \equiv D) \longrightarrow \begin{cases} T(C \sqsubseteq D) \\ T(D \sqsubseteq C) \end{cases}$$

As we saw in Section 4.2.1, range and domain axioms  $\top \sqsubseteq \forall P.D$  and  $\top \sqsubseteq \forall P^-.D$  are mapped into def-Horn rules of the form  $D(y) \leftarrow P(x, y)$  and  $D(x) \leftarrow P(x, y)$  respectively. Moreover, class-instance and instance-property-instance axioms  $a : D$  and  $\langle a, b \rangle : P$  are mapped into def-Horn facts (i.e., rules with empty bodies) of the form  $D(a)$  and  $P(a, b)$  respectively. We therefore extend  $T$  to deal with these axioms in the case that  $D$  is an  $\mathcal{L}_h$ -class:

$$\begin{aligned} T(\top \sqsubseteq \forall P.D) &\longrightarrow Th(D, y) \leftarrow P(x, y) \\ T(\top \sqsubseteq \forall P^-.D) &\longrightarrow Th(D, x) \leftarrow P(x, y) \\ T(a : D) &\longrightarrow Th(D, a) \\ T(\langle a, b \rangle : P) &\longrightarrow P(a, b) \end{aligned}$$

where  $x, y$  are variables and  $a, b$  are constants.

Finally, we extend  $T$  to deal with the property axioms discussed in Section 4.2:

$$\begin{aligned} T(P \sqsubseteq Q) &\longrightarrow Q(x, y) \leftarrow P(x, y) \\ T(P \equiv Q) &\longrightarrow \begin{cases} Q(x, y) \leftarrow P(x, y) \\ P(x, y) \leftarrow Q(x, y) \end{cases} \\ T(P \equiv Q^-) &\longrightarrow \begin{cases} Q(x, y) \leftarrow P(y, x) \\ P(y, x) \leftarrow Q(x, y) \end{cases} \\ T(P^+ \sqsubseteq P) &\longrightarrow P(x, z) \leftarrow P(x, y) \wedge P(y, z) \end{aligned} \quad (1)$$

**Definition 1 (Description Horn Logic)** A Description Horn Logic (DHL) ontology is a set of DHL axioms of the form  $C \sqsubseteq D$ ,  $A \equiv B$ ,  $\top \sqsubseteq \forall P.D$ ,  $\top \sqsubseteq \forall P^-.D$ ,  $P \sqsubseteq Q$ ,  $P \equiv Q$ ,  $P \equiv Q^-$ ,  $P^+ \sqsubseteq P$ ,  $a : D$  and  $\langle a, b \rangle : P$ , where  $C$  is an  $\mathcal{L}_b$ -class,  $D$  is an  $\mathcal{L}_h$ -class,  $A, B$  are  $\mathcal{L}$ -classes,  $P, Q$  are properties and  $a, b$  are individuals.

Using the relationships of (full) DL to FOL discussed in Section 3.1, especially Figure 4, it is straightforward to show the following.

**Theorem 1 (Translation Semantics)** The mapping  $T$  preserves semantic equivalence. Let  $\mathcal{K}$  be a DHL ontology and  $\mathcal{H}$  be the def-Horn ruleset that results from applying the mapping  $T$  to all the axioms in  $\mathcal{K}$ . Then  $\mathcal{H}$  is logically equivalent to  $\mathcal{K}$  w.r.t. the semantics of FOL —  $\mathcal{H}$  has the same set of models and entailed conclusions as  $\mathcal{K}$ .

DHL can, therefore, be viewed alternatively and precisely as an expressive fragment of def-Horn— i.e., as the *range* of  $T$  (DHL).

#### 4.5 Expressive Power of DHL

Although the asymmetry of DHL (w.r.t. classes on different sides of axioms) makes it rather unusual by DL standards, it is easy to see that it includes (the DAML+OIL subset of) RDFS, as well as that part of DAML+OIL which corresponds to a simple frame language.

As far as RDFS is concerned, we saw in Section 4.2.1 that RDFS statements are equivalent to DL axioms of the form  $C \sqsubseteq D$ ,  $\top \sqsubseteq \forall P.C$ ,  $\top \sqsubseteq \forall P^-.C$ ,  $P \sqsubseteq Q$ ,  $a : D$  and  $\langle a, b \rangle : P$ , where  $C, D$  are classes,  $P, Q$  are properties and  $a, b$  are individuals. Given that all RDFS classes are  $\mathcal{L}$ -classes (they are atomic class names), a set of DL axioms corresponding to RDFS statements would clearly satisfy the above definition of a DHL ontology.

DHL also includes the subset of DAML+OIL corresponding to simple frame language axioms, i.e., axioms defining a primitive hierarchy of classes, where each class is defined by a frame. A frame specifies the set of subsuming classes and a set of slot constraints. This corresponds very neatly to a set of DL axioms of the form  $A \sqsubseteq \mathcal{L}_h$ .

Moreover, DHL supports the extension of this language to include equivalence of conjunctions of atomic classes, and axioms corresponding to DAML+OIL transitive property, and inverse property statements.

## 5. DEFINING DLP

**Definition 2 (Description Logic Programs)** *We say that a def-LP  $\mathcal{RP}$  is a Description Logic Program (DLP) when it is the LP-correspondent of some DHL ruleset  $\mathcal{RH}$ .*

A DLP is directly defined as the LP-correspondent of a def-Horn ruleset that results from applying the mapping  $\mathcal{T}$ . Semantically, a DLP is thus the f-weakening of that DHL ruleset (recall subsection 3.2). The DLP expressive class is thus the expressive f-subset of DHL. By Theorem 1, DLP can, therefore, be viewed alternatively and precisely as an expressive subset of DL, not just of def-Horn.

In summary, expressively DLP is contained in DHL which in turn is contained in the expressive intersection of DL and Horn.

## 6. MORE ABOUT TRANSLATING

As our discussion of expressive relationships has made clear, there is a bi-directional semantic equivalence of (1) the DHL fragment of DL and (2) the DHL fragment of def-Horn. Likewise, there is a bi-directional semantic equivalence of the DLP fragment of DL and the DLP fragment of def-Horn. So far, however, we have mostly concentrated on only one direction of *syntactic* mapping: from DL syntax to def-Horn syntax (and to the corresponding def-LP), rather than from def-Horn (or def-LP) to DL. Next, we elucidate our reasons for this emphasis.

First, a prime immediate goal for the Semantic Web is to enable rules (in LP / Horn) on top of ontologies (in DL) — more than vice versa to enable DL ontologies on top of LP or Horn rules. Second, it is desirable to exploit the relatively numerous, mature, efficient, scalable algorithms and implementations (i.e., engines) already available for LP inferencing so as to perform some fragment of DL inferencing — more than vice versa to perform LP via the fewer available DL engines, which are designed to handle more expressive languages (than DLP) and may not be optimised for DLP ontologies. Third, as compared to def-Horn, DL has a relatively detailed set of quite specific syntactic expressive constructs; it was easier to go through these one by one to define a translation mapping than to do so in the reverse direction where one has to invent more structure/forms.

We do not have space here to give detailed algorithms and computational complexity analyses of the syntactic translations. We will limit ourselves to some relatively high-level observations; these are straightforward to show. The  $\mathcal{T}$  mapping, from DL syntax to def-Horn/def-LP syntax, corresponds immediately to an al-

gorithm whose computational complexity is tractable. This mapping is invertible (e.g., in the usual manner of parsers) from def-Horn/def-LP syntax to DL syntax, again, tractably.

## 7. INFERENCE

As discussed in the previous section, one of the prime goals of this work is to enable some fragment of DL inferencing to be performed by LP engines. In this section we will discuss the kinds of inference typically of interest in DL and LP, and how they can be represented in each other, i.e., in LP and DL respectively. Although the emphasis is on performing DL inferencing, via our mapping translation, using an LP reasoning engine, the reverse mapping can be used in order to perform LP inferencing using a DL reasoning engine. In particular, we will show how inferencing in (the DHL fragment of) DL can be reduced, via our translation, to inferencing in LP; and how vice versa, inferencing in (the DLP fragment of) LP can be reduced to inferencing in DL.

In a DL reasoning system, several different kinds of query are typically supported w.r.t. a knowledge base  $\mathcal{K}$ . These include queries about classes:

1. class-instance membership queries: given a class  $C$ ,
  - (a) ground: determine whether a given individual  $a$  is an instance of  $C$ ;
  - (b) open: determine all the individuals in  $\mathcal{K}$  that are instances of  $C$ ;
  - (c) “all-classes”: given an individual  $a$ , determine all the (named) classes in  $\mathcal{K}$  that  $a$  is an instance of;
2. class subsumption queries: i.e., given classes  $C$  and  $D$ , determine if  $C$  is a subclass of  $D$  w.r.t.  $\mathcal{K}$ ;
3. class hierarchy queries: i.e., given a class  $C$  return all/most-specific (named) superclasses of  $C$  in  $\mathcal{K}$  and/or all/most-general (named) subclasses of  $C$  in  $\mathcal{K}$ ;
4. class satisfiability queries, i.e., given a class  $C$ , determine if  $C$  is satisfiable (consistent) w.r.t.  $\mathcal{K}$ .

In addition, there are similar queries about properties: property-instance membership, property subsumption, property hierarchy, and property satisfiability. We will call  $\mathcal{QDL}$  the language defined by the above kinds of DL queries.

In LP reasoning engines, there is one basic kind of query supported w.r.t. a ruleset  $\mathcal{R}$ : atom queries. These include:

1. ground: determine whether a ground atom  $A$  is entailed;
2. open (ground is actually a special case of this): determine, given an atom  $A$  (in which variables may appear), all the tuples of variable bindings (substitutions) for which the atom is entailed.

We call  $\mathcal{QLP}$  the language defined by the above kinds of LP queries.

Next, we discuss how to reduce  $\mathcal{QDL}$  querying in (the DHL fragment of) DL to  $\mathcal{QLP}$  querying in (the DLP fragment of) LP using the mapping  $\mathcal{T}$ . We will assume that  $\mathcal{R}$  is a ruleset derived from a DL knowledge base  $\mathcal{K}$  via  $\mathcal{T}$ , and that all  $\mathcal{QDL}$  queries are w.r.t.  $\mathcal{K}$ .

$\mathcal{QLP}$  (ground or open) atom queries can be used to answer  $\mathcal{QDL}$  (ground or open) class-instance membership queries when the class is an  $\mathcal{L}_h$ -class, i.e.,  $a$  is an instance of  $C$  iff  $\mathcal{R}$  entails

$\mathcal{T}(a : C)$ . When  $C$  is an atomic class name, the mapping leads directly to a  $\mathcal{QLP}$  atom query. When  $C$  is a conjunction, the result is a conjunction of  $\mathcal{QLP}$  atom queries, i.e.,  $a$  is an instance of  $C \sqcap D$  iff  $\mathcal{R}$  entails  $\mathcal{T}(a : C)$  and  $\mathcal{R}$  entails  $\mathcal{T}(a : D)$ . When  $C$  is a universal restriction, the mapping  $\mathcal{T}(a : \forall P.C)$  gives  $\mathcal{T}(C, y) \leftarrow P(a, y)$ . This can be transformed into a  $\mathcal{QLP}$  atom query using a simple kind of skolemisation, i.e.,  $y$  is replaced with a constant  $b$ , where  $b$  is new in  $\mathcal{R}$ , and we have  $a$  is an instance of  $\forall P.C$  iff  $\mathcal{R} \cup \{P(a, b)\}$  entails  $\mathcal{T}(b : C)$ .

The case of property-instance membership queries is trivial as all properties are atomic:  $\langle a, b \rangle$  is an instance of  $P$  iff  $\mathcal{R}$  entails  $P(a, b)$ .

Complete information about class-instance relationships, to answer open or “all-classes” class-instance queries, can then be obtained via class-instance queries about all possible combinations of individuals and classes in  $\mathcal{K}$ .<sup>9</sup> (Note that the set of named individuals and classes is known, and its size is worst-case linear in the size of the knowledge/rule base.)

For  $\mathcal{L}_h$ -classes,  $\mathcal{QDL}$  class subsumption queries can be reduced to  $\mathcal{QLP}$  using a similar technique to class-instance membership queries, i.e.,  $C$  is a subclass of  $D$  iff  $\mathcal{R} \cup \{\mathcal{T}(a : C)\}$  entails  $\mathcal{T}(a : D)$ , for  $a$  new in  $\mathcal{R}$ . For  $\mathcal{QDL}$  property subsumption queries,  $P$  is a subproperty of  $Q$  iff  $\mathcal{R} \cup P(a, b)$  entails  $Q(a, b)$ , for  $a, b$  new in  $\mathcal{R}$ .

Complete information about the class hierarchy can be obtained by computing the partial ordering of classes in  $\mathcal{K}$  based on the subsumption relationship.

In the DHL (and DLP) fragment, determining class/property satisfiability is a non-issue as, with the expressive power at our disposal in def-Horn, it is impossible to make a class or a property unsatisfiable.

Now let us consider the reverse direction from  $\mathcal{QLP}$  to  $\mathcal{QDL}$ . In the DLP fragment of LP, every predicate is either unary or binary. Every atom query can thus be viewed as about either a named class or a property. Also, generally in LP, any open atom query is formally reducible to a set of ground atom queries—one for each of its instantiations. Thus  $\mathcal{QLP}$  is reducible to class-instance and property-instance membership queries in DL.

To recap, we have shown the following.

**Theorem 2 (Inferencing Inter-operability)** For  $\mathcal{L}_h$ -classes,  $\mathcal{QDL}$  querying in (the DHL fragment of) DL is reducible to  $\mathcal{QLP}$  querying in (the DLP fragment of) LP, and vice versa.

## 8. TRANSLATION TO DATABASES

Luckily, Logic Programming is also an elegant language for data-oriented problems, for example it allows one to obtain languages equivalent to known database languages by making various syntactic restrictions. One language that can be obtained by such restrictions is Datalog, which underlies deductive databases, and closely corresponds to the def-Horn subset of FOL introduced in Section 3.

### 8.1 Translation to relational databases

Datalog programs can be implemented on top of relational databases. To perform this implementation all explicit facts of a predicate  $P$  are stored in a dedicated table  $P_{Ext}$ . All non-recursive rules are translated to relational views. Rule bodies are translated to appropriate SQL queries (usually operating on other views). To obtain all explicit and implicit information, a view is defined to

<sup>9</sup>More efficient algorithms would no doubt be used in practise.

represent each predicate  $p$ . The query of the view integrates the explicit information, found in  $p_{Ext}$  with the queries that represent the bodies of those rules where  $p$  is the head. The interested reader may refer to [18] for an in-depth description, algorithm and proof. Intuitively, this result follows from the following substitutions: each Datalog-rule can be simulated using the select-from-where construct of SQL; multiple rules defining the same predicate can be simulated using *union*; and negation in rule bodies can be simulated using *not in*.

To compute the answer for user queries the translated views are used. This realises a form of bottom up processing, since the queries involved in view definitions are performed on the extensional data and intermediate results are propagated up to a final query, which is the user query. This results in many irrelevant facts being computed in the intermediate steps; more efficient procedures based on sideways information passing have, however, been developed in the deductive database literature.

The above mentioned strategy is, however, not possible for recursively defined rules. Here additional processing is required.

### 8.2 Handling recursion

Modern relational database systems, which support the SQL:99 standard, can process some limited form of recursion, namely linear recursion with a path length one. Hence, the predicate used as the rule head may occur only once in the rule body. Cycles other than such linear self-references can also not be implemented.

Usually, binary recursive rules such as transitivity can be rewritten into a linear form. E.g. the mapping for transitive properties (see 1) can be rewritten into

$$\begin{aligned} P(x, y) &\leftarrow P_{Ext}(x, y). \\ P(x, z) &\leftarrow P_{Ext}(x, y) \wedge P(y, z). \end{aligned}$$

The usual strategy to compute the remaining forms of recursive rules in relational databases is in-memory processing using some iterative strategy, e.g. the magic template procedure [15].

*Indirect Recursion.* The remaining cases of non-linear recursion that cannot be rewritten into the SQL:99 constructs are mainly represented by the possibility of having cyclical class and property hierarchies.

We can, however, translate this case into the database by exploiting the observation that this form of recursion decomposes into unions, since no join processing of intermediate results (such as involved in computing the transitive closure of transitive properties) is necessary. This is immediately clear for classes, since they are monadic predicates. A closer look at all axioms where binary predicates (properties) are in the head reveals the same. Hence, these cyclic references can be implemented via an algorithm that detects equivalence classes (each constituted by a cycle) in graphs. All incoming edges to an equivalence class must be duplicated to all members of the equivalence class; this may done by using a new intermediate predicate to collect the incoming edges and deriving the members of the equivalence class from this intermediate predicate. Afterwards, all rules that constitute the cyclic references within the equivalence class may safely be removed. The reader may note that this can also be performed (with appropriate adaptations) on the cyclic references imposed by inverse properties.

### 8.3 Implementation

We have used the above techniques to realise a prototypical implementation of Description Horn Logic based on the Datalog engine written by Boris Motik in the KAON project. The implementation, called Bubo (after the Latin name of the biological genus of

eagle owls), is freely available at <http://kaon.semanticweb.org/owl/>.

Initial tests of Bubo have been encouraging, but much more work needs to be done in order to determine if the benefits promised by the DLP-fusion approach are delivered by this implementation.

## 9. DISCUSSION

In this paper we have shown how to interoperate, semantically and inferentially, between the leading Semantic Web approaches to rules (RuleML Logic Programs) and ontologies (OWL/DAML+OIL Description Logic). We have begun by studying two new KR's, Description Logic Programs (DLP), which is defined by the expressive intersection of the two approaches, and the closely related Description Horn Logic (DHL).

We have shown that DLP (or DHL) can capture a significant fragment of DAML+OIL, including the whole of the DAML+OIL fragment of RDFS, simple frame axioms and more expressive property axioms. The RDFS fragment of DL permits: stating that a class *D* is a *Subclass* of a class *E*; stating that the *Domain* of a property *P* is a class *C*; stating that the *Range* of a property *P* is a class *C*; stating that a property *P* is a *Subproperty* of a property *Q*; stating that an individual *b* is an *Instance* of a class *C*; and stating that a pair of individuals (*a,b*) is an *Instance* of a property *P*. Additional DLP expressively permits (within DL): using the *Intersection* connective (conjunction) within *class* descriptions (i.e., in *C*, *D*, or *E* above); using the *Union* connective (disjunction) within *subclass* descriptions (i.e., in *D* above); using (a restricted form of) *Universal* quantification within *superclass* descriptions (i.e., in *E* above); using (a restricted form of) *Existential* quantification within *subclass* descriptions (i.e., in *D* above); stating that a property *P* is *Transitive*; stating that a property *P* is *Symmetric*; and stating that a property *P* is the *Inverse* of a property *Q*.

Many of the ontologies in the DAML ontology library are inside this fragment of DAML+OIL. An immediate result of this work is that LP engines could be used for reasoning with these ontologies and for reasoning with (possibly very large numbers of) facts, such as web page annotations, that use vocabulary from these ontologies.

This work represents only a first step in realising a more complete interoperability between rules and ontologies, and the layering of rules on top of ontology languages in the Semantic Web "stack". We were able to illustrate its utility both theoretically and within our prototypical implementation. We believe, however, that our study of the expressive intersection will provide a firm foundation for future investigations of more expressive languages up to and including the expressive union of rules and ontologies.

Future work will include extending the mapping to additional DL primitives, in particular those which require the ability to state and derive the equality of individuals, such as cardinality restrictions (including functional properties) and nominals (extensionally defined classes).

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