

# Information Structures, Price Discrimination and Demand Uncertainty

Adib Bagh (*abagh@ucdavis.edu*) and Hemant K. Bhargava (*hemantb@ucdavis.edu*)

Numerous contract scenarios in information technology goods and services, telecommunications, digital entertainment, and supply chain contracting feature demand uncertainty for individual customers (demand is subject to idiosyncratic demand shocks that change marginal valuations). Sellers in these scenarios often use non-linear pricing mechanisms (flat rate, two part tariffs, three-part tariffs, block-declining tariffs) in order to price discriminate between heterogeneous consumers. These contracts typically cover a long time, and sellers face a choice about the degree of information they allow the consumers to have about their demand when choosing terms from the contract menu. For example, the seller could ask a consumer to choose from a non-linear price menu at the beginning of the contract period (when the buyer’s demand is still uncertain); such *ex-ante* contracts occur in sales force compensation, “push” supply contracts between firms, and many online hosting services. Alternately, the seller could allow the customer to choose the terms of contract *ex-post* after realization of the demand shock (this realization is private information to the buyer). In this case, the only *ex-ante* decision made by customers is whether or not to sign the contract. The seller, on the other hand, must set the contract terms without knowledge of the demand shock. A striking example of this is Nextel’s *Auto Adjust* wireless calling plans where, given a menu of plan levels and prices, the contract “automatically adjusts to put you on the lowest plan level based on how much you used your phone.” Other examples are “pull” supply contracts, buyback policies, and wireless calling plans which allow consumers switch their current plan at month-end in order to get a lower total price for that month. Figure 1 illustrates two extremes of information structures.

We are interested in the different factors that affect a monopolist’s decision regarding her choice of information structure to impose on buyers. On one hand, forcing the buyers to sign a contract before they resolve idiosyncratic shocks allows the seller to extract entire surplus (minus any information rents to induce self-selection) of the average buyer. But, could the seller extract additional surplus by allowing buyers to resolve demand uncertainty before choosing the terms of a contract? Since the resolution of uncertainty creates a better match between buyers preferences and their consumption patterns, the seller could with some probability—and by using an appropriate pricing mechanism—extract some of

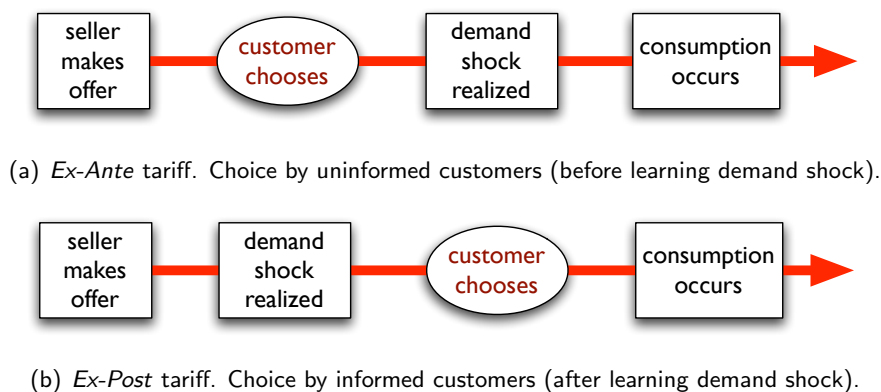


Figure 1: Two information structures for offering a tariff.

this additional surplus. Clay et al. (1992) showed that the net expected gain depends on the size of demand shock (*ex-post* pricing increases profit when the shock is large), and Miravete (1996) showed that the effect depends both on the degree of uncertainty and the extent of consumer heterogeneity. Both sets of results were obtained in the context of two-part tariffs. Our key result shows that the net effect also depends on the type of tariff structure, hence the monopolist's problems of choosing the type of (non-linear) tariff and information structure of the contract are not separable. We also show that in many cases, a single three-part tariff—a simpler mechanism than a menu of two-part tariffs—can beat a menu of two-part tariffs, whether offered *ex-ante* or *ex-post*.

## 1 Tariff Structure under Deterministic demand

We start our analysis with the case where customers have deterministic demand. Our results in this section will serve as a tool that will help us analyze the case of stochastic demand. We assume there are two customer types  $i = 1, 2$  with demand functions  $D_i(p)$ , where  $D_1$  is the low demand and  $D_2$  the high (assuming the standard single-crossing property). Let  $\lambda$  and  $1 - \lambda$  denote the proportions of types 1 and 2 in the population. Customers know their demand precisely, while the firm knows only the distribution of types and the demand function conditional on type.

### 1.1 Menu of two-part tariffs

Menus of tariffs are used by firms in order to segment a market of heterogenous consumers by inducing self selection among these consumers. We consider the case where a firm offers a menu of two-part tariffs,  $[(F_1, p_1); (F_2, p_2)]$ , the  $F_i$ s are fixed fees, and  $p_i$ s are unit fees for marginal consumption. With the optimal menu, type 1's surplus is extracted by setting  $F_1 = S_1(p_1)$ . The fixed fee  $F_2$  for type 2 is set to ensure self-selection. All offers must meet the individual-rationality constraints. We assume it is always optimal for the firm to sell to type 1 (the low type).

**Lemma 1 (Optimal menu of two 2-part tariffs)** *The optimal menu  $[(F_1^*, p_1^*); (F_2^*, p_2^*)]$  has*

$$p_1^* = \text{sol.} \left[ \frac{(1 - \lambda)}{\lambda} = \frac{-(p_1 - c)D_1'(p_1)}{(D_2(p_1) - D_1(p_1))} \right] \quad (1)$$

$$F_1^* = \int_{p_1^*}^{\infty} D_1(\tau) d\tau \quad (2)$$

$$p_2^* = c \quad (3)$$

$$F_2^* = F_1^* + \int_c^{p_1^*} D_2(\tau) d\tau \quad (4)$$

In the case of linear demand, Eq. 1 can be solved explicitly for  $p_1^*$ .

### 1.2 Three-part tariff

We consider an optimal single three-part tariff (3PT)  $(\hat{F}, \hat{Q}, \hat{p})$  for the case of two consumers with deterministic demand.  $\hat{F}$  is a fixed fee that covers up to  $\hat{Q}$  units, and  $\hat{p}$  is the price per unit in excess of  $\hat{Q}$ . The following result summarizes the optimal 3PT in the case of two customers with demand functions  $D_i(p)$  for  $i \in 1, 2$ .

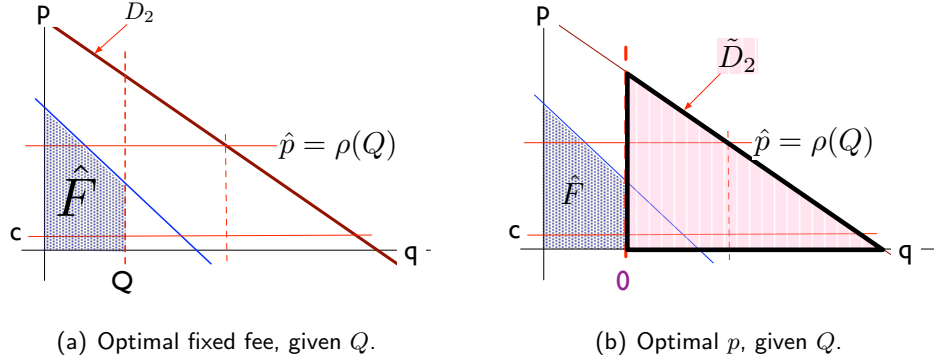


Figure 2: Design of optimal three-part tariff.

**Lemma 2 (Optimal three-part tariff)** *The optimal three-part tariff  $(\hat{F}, \hat{Q}, \hat{p})$  is*

$$\hat{F} = \int_0^{\hat{Q}} D_1(\tau) d\tau \quad (5)$$

$$\hat{p} = \rho(\hat{Q}) = \arg \max_p (p - c)(D_2(p) - \hat{Q}) \quad (6)$$

$$\hat{Q} = \arg \max_Q \hat{F} + (1 - \lambda)(\hat{p} - c)(D_2(\hat{p}) - Q)$$

$$= \arg \max_Q \left[ \int_0^Q D(\tau) d\tau + (1 - \lambda) \max_p ((p - c)(D_2(p) - Q)) \right] \quad (7)$$

The linear demand case ( $D_i(p) = \frac{1}{b_i}(a_i - p)$ ) yields

$$\hat{p} = \frac{c}{2} + \frac{a_2 - b_2 \hat{Q}}{2}$$

$$\hat{F} = \hat{Q} \left( (a_1 - b_1 \hat{Q}) + \frac{b_1 \hat{Q}}{2} \right)$$

$$\hat{Q} = \frac{a_1 - (1 - \lambda)a_2}{b_1 - (1 - \lambda)b_2}$$

First, note that it is optimal to set the tariff components so that the low type has no consumption in excess of the allowance  $Q$ . The best  $F$  corresponding to  $Q$  is type 1's entire surplus on consuming  $Q$  units (see Fig. 2(a)). Both types get  $Q$  "free" units for this fixed fee. The firm should choose  $p$  to extract maximum surplus from type 2 as shown in Fig. 2(b). This optimization uses the truncated demand function  $\tilde{D}_2(p, Q) = (D_2(p) - Q)$  to take into account the first  $Q$  units of consumption. Solving this optimization problem, let  $\hat{p}$  be some function  $\rho(Q)$ . Putting everything together, the optimal 3PT is obtained by choosing  $Q$  that maximizes profit.

### 1.3 Single three-part tariff beats menu of two-part tariffs

The menu of two-part tariffs is designed to exploit the heterogeneity between the two customer segments (the menu may offer up to  $I$  choices where  $I$  is the number of customer types). A single three-part

tariff seemingly reduces the seller's flexibility since it assigns the same fixed fee  $F$  and allowance  $Q$  to both types, and the only difference is in the additional revenue from type 2's overflow consumption. Surprisingly, this "simpler" design can yield higher profit. The (single) three-part tariff actually gives the seller greater flexibility in segmenting the two customer types by eliminating the incentive compatibility constraint which holds down surplus extraction in the menu of two-part tariffs.

**Theorem 1** *Under deterministic demand functions of the form  $D_i = (a_i - bp)$ ,  $i \in 1, \dots, I$  where  $a_1 > a_2 > \dots > a_I$ , there exists  $\hat{\lambda}(a_1, a_2, b)$  such that if  $\frac{\lambda_1}{\lambda_i} > \hat{\lambda}$ , for  $\lambda_i \neq 1$ , then a single three-part tariff yields higher profit than the optimal two-part tariff menu consisting of  $I$  items.*

If  $I = 2$  and  $b = 1$  (slope of demand function), then  $\hat{\lambda} = \sqrt{0.5}$ . This threshold value is satisfied in many markets because the ratio of the low-type (or "light") users is far larger than the ratio of high-type (or "heavy") users. With multiple customer types ( $I > 2$ ), the requirement is analogous: the lowest type must constitute a relatively large portion of the population. Theorem 1 is especially significant in light of Miravete (2004)'s empirical finding that, often, firms "over design" their non-linear price mechanism, choosing a large menu that yields only a small incremental gain over a more compact design. Theorem 1 makes a stronger statement: not only is the three-part tariff simpler, but it can be more profitable. With a three-part tariff, we can use  $F$  and  $Q$  to extract from the low type at least the same amount of surplus that we can extract using a two-part tariff. Moreover,  $p$  will allow us to extract from the high type more surplus than we can extract using two-part tariffs.

## 2 Tariff and Information Structure under Demand Uncertainty

Our specification of demand uncertainty and timing of agent actions follows Clay et al. (1992). There are two customer types with demand functions  $D_i(p, \tilde{Z})$  where  $\tilde{Z}$  is a random variable that can take two values, low and high. Specifically, writing  $\bar{D}$  for expected demand, the demand realization  $D_i$  can be either  $D_i^L = \bar{D}_i - Z_i$  (or low demand) and  $D_i^H = \bar{D}_i + Z_i$  (or high demand), with equal probability. Upon realization, the type  $i$  consumer becomes aware of her type, but the firm remains uninformed.  $Z_i$  is a measure of the degree of the uncertainty.

Under the two-part tariff scheme, the *ex-ante* problem for the seller is independent of the uncertainty level  $Z_i$ . The optimal two-part menu is then obtained by following the procedure of §1.1. With *ex-post* pricing, the firm faces up to four customer types, and profit is maximized over the *ex-post* distribution of types. This problem is equivalent to, *ex-post*, offering the customers (four customers if no pooling is needed, less than four if pooling is optimal) a taper or a declining-block tariff. Clay et al. showed that using two-part menus, *ex-ante* contracts generate higher profits than *ex-post* contracts when the demand uncertainty is low ( $Z_i$  is less than some  $\hat{Z}$ ). When the demand uncertainty is high ( $Z_i > \hat{Z}$ ) and still using menus of two-part tariffs, *ex-post* pricing generate more profits than *ex-ante* pricing.

For a single tariff, the only choice the consumer faces *ex-ante* is whether or not to sign the contract. Suppose first that we only shock the high type. Then, we obtain the following result

**Theorem 2** *Consider the case when only the demand of the high type is uncertain. When the level of uncertainty is high ( $Z_i > \hat{Z}$  threshold), a three-part tariff offered *ex-post* is the most profitable among all structures. Under low uncertainty ( $Z_i < \hat{Z}$ ), the single three-part tariff is more profitable than a menu of two-part tariffs, whether offered to uninformed or informed customers, when the low-type customers are a substantial fraction of the population.*

When the high type has a small demand shock, the optimal 3PT  $(\hat{F}, \hat{Q}, \hat{p})$  yields the same profit whether it is offered *ex-ante* or *ex-post* (when the shock is small enough to prevent the *ex-post* low demand from being unattractive to serve or to pool with low type's demand). When the high type has a high demand shock level, however, the seller can improve his profit by only targeting the *ex-post* types with demands  $D_i^H$ . The result of Theorem 2 remains valid when a demand shock also hits the low-type. The only difference is that in this case an *ex-post menu* of three-part tariffs will be the optimal choice.

Besides stating that a single 3PT offered *ex-post* is often a better tariff design, Theorem 2 also highlights that the choice of information structure is inseparable from the choice of tariff structure when buyers have uncertain demand. Finally, we remark that an *ex-post* menu of three-part tariffs is a type of piece-wise linear, multi-block tariff. Therefore, there is an indication that at least in the case of low levels of demand uncertainty, using a non-concave piece-wise linear tariff can be more profitable than the more commonly used concave or declining block tariff. In fact, the Auto-Adjust plan by Nextel that we mentioned in the introduction can be presented by a non-concave and piece-wise linear tariff.

### 3 Conclusion

The literature on price discrimination traditionally takes the information structure in the market as given. Under demand uncertainty, however, a monopolist not only has control over the type of payment plans in the contract, she also has control over the time line of the contract. Markets for telecommunication and information technologies are rife with examples of different types of contracts that target customers with various degrees of uncertainty in their demands. Our results are influenced by the *combination* of demand uncertainty and the heterogeneity between types in the model. More specifically, we showed that for various levels of demand uncertainty, the seller will benefit from using a three-part tariff and from allowing the consumers to choose a payment plan ( the terms of the contract) *after* their demand uncertainty is resolved. The use of three-part tariff allows the seller to reduce the size of her menu (the number of items on the menu), and to simplify the comparison between offering the *ex-ante* and *ex-post* pricing. Our analysis was restricted to three-part tariffs (which encompass linear pricing, two-part tariffs, and flat rates) two consumer types, and symmetric demand shocks. The same analysis can be carried for markets with more consumer types. For such markets, the seller can use larger menus ( menus with more items) of three-part tariffs. Allowing for more general demand shocks is possible but will considerably complicate the model.

### References

- CLAY, K. B., D. S. SIBLEY, AND P. SRINAGESH (1992): "Ex Post vs. Ex Ante Pricing: Optional Calling Plans and Tapered Tariffs," *Journal of Regulatory Economics*, 4, 115–38, available at <http://ideas.repec.org/a/kap/regeco/v4y1992i2p115-38.html>.
- MIRAVETE, E. J. (1996): "Screening Consumers Through Alternative Pricing Mechanisms," *Journal of Regulatory Economics*, 9, 111–132.
- (2004): "Are all those Calling Plans Really Necessary? The Limited Gains From Complex Tariffs," Tech. Rep. 4237, CEPR Discussion Papers, available at <http://ideas.repec.org/p/cpr/ceprdp/4237.html>.